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The cave resonator and the Parker Turner cave collapse problem

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ABSTRACT

The Parker Turner cave diving accident was unique. Out of several hundred cave diving fatalities that occurred in Florida during the past 50 years, it is the only one that occurred due to a partial collapse of the cave. Here, we propose a natural physical process that might explain this unique accident.

While it is possible that the presence of the divers in the cave while the cave collapsed just happened to occur simultaneously, this is unlikely. It is suggested here that resonance in the air pockets in the cavern (or cave), created by breathing (open circuit) divers, may have contributed to the collapse. We propose that divers present in the cavern during the dive may have (unknowingly) caused the collapse through the pressurized air that they release with each breath. When the breathing period of the diver(s) matches the natural oscillations period of our new "cave oscillator", the ensuing resonance causes the air pressure in the pockets to increase uncontrollably.

We model the system as a non-uniform *U*-tube filled with water on the bottom and compressed air on top. The top of the tube is sealed on both sides so that the compressed air is trapped in the chambers above the water. The bottom part of the tube represents the water filled cavern (or cave) whereas the vertical tubes represent the air cavities. We show analytically that such a system is subject to natural oscillations with a period of roughly the same as the breathing rate of the typical diver.

1. Introduction

Indian Spring (latitude 30° 15.082, and longitude 84° 19.317) is a fresh water spring located in Wakulla County, Florida (Figs. 1 and 2). It is part of a hidden underworld known as the Woodville Karst Plain that stretches over 1000 square kilometers from just south of Tallahassee into the Gulf of Mexico. The region covers the longest known underwater cave system in the United States (∼30 km). Indian Spring cave is approximately 30 m deep and its length is unknown; a combined distance of about 3 km has been surveyed (by divers) upstream and downstream of the cave. The spring "run" (i.e., the part of the flow that is above ground) flows for about a kilometer southeast and empties into Sally Ward Spring at the headwaters of the Wakulla River. The Indian spring discharge is highly variable but is estimated to be several cubic meters per second (see e.g., Schmidt (2004) for a description of regional springs).

Like most Florida springs, Indian Spring contains a cavern (defined as the portion of the cave that is usually reached by sunlight) and a cave (defined as a narrow lightless tube) and Fig. 3 displays a typical cross section of such a cave. The Florida aquifer consists of porous limestone that dissolves in freshwater, forming a "cheese-like" medium. Part of the medium is just like any other porous media but, unlike the familiar porous mediums, tubes and tunnels are embedded in it. The typical cave usually "snakes" around leaving concave portions that often contain pockets of compressed air released by cave divers. Cave divers typically use breathing gas mixtures different from air but, for the sake of simplicity, we will use the term "gas" or "air" interchangeably for the various gaseous mixture. An interesting related question that comes immediately to mind is whether the composition of the exhaled breathing gas affects the limestone in the air pockets to the extend that regular, frequent diving weaken the limestone in these concave areas and increase the risk of collapse. We do not know how to estimate this possibility with the tools that we presently have.

Very strong flows occasionally flush the pockets out but new ones are formed when the flow subsides. Also, in most caves, the gas in the pockets gradually percolates upward through the porous limestone. Despite both processes, however, gas pockets are found in most cave dives indicating that they are present most of the time. Although we shall focus on Indian Springs where the particular accident in question occurred, the conditions in that cave as well as its structure are not that unique. The resonance that we propose may well also apply to other caves in limestone (e.g., aquifers in Florida, the Yucatan and the Karst regions in central Europe), ice caves, and perhaps even wrecks.

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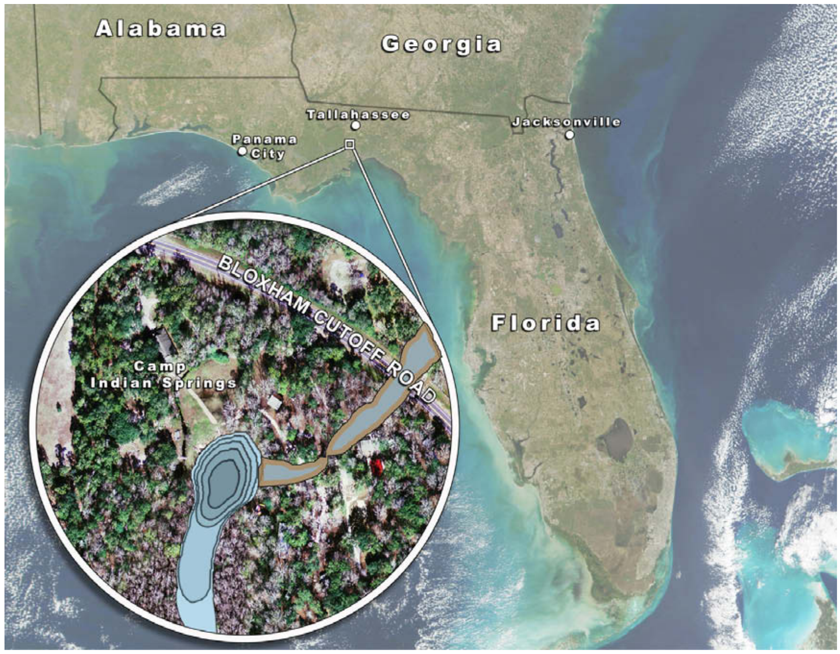

Fig. 1. A map of the southeastern United States showing the location of the water-filled Indian Spring cave. The cave spring water, which flows at a speed ∼$O(0.1\ \text{m/s})$ within the cave, ultimately ends up in the spring pool which eventually empties into the Wakulla river. Figure adapted from larger NASA (http://visibleearth.nasa.gov/view_rec.php?id=2408) and Labins (http://data.labins.org) website maps.

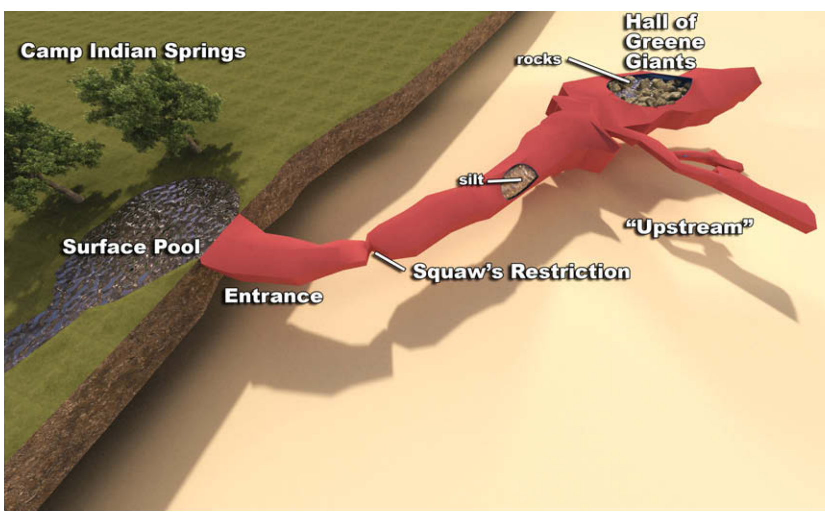

Fig. 2. A schematic three-dimensional view of the Indian Spring cave. The cave walls, ceiling and floor are all shown with the red envelope. Limestone is outside that envelope whereas water occupies the inside of the envelop (the cave). Our proposed collapse mechanism involves resonance in the region downstream to the Squaws Restriction. That resonance increased the pressure in divers-induced air pockets (in the cavern region) beyond the level that which the cavern's ceiling could sustain. As a result, part of the ceiling fell on the steeply sloped sediment, which, in turn, slid and blocked the Squaws Restriction (see Figs. 3–5). Figure created by Conn & Associates Architects, Inc., Tallahassee, FL. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

What is reported here is what one may term "forensic environmental fluid dynamics", which is a blend of forensic study and an investigation of the fluids stability within the cave. This is not a forensic study per se, however, because: (i) the emphasis here is on the stability of the fluid system, and (ii) much of the accident reporting is anecdotal. We attempted to be much more quantitative with regard to item (ii) above. However, the event has been very traumatic to many of those involved and has generated much friction among them. As a result, some have quit cave diving all together and those who did not quit do not wish to speak about it. So much so, that all of our continuous and persistent requests to speak about the accident to those that were present in the cave and cavern fell on deaf ears. This unfortunate (but understandable) situation results from the fact that the agencies associated with cave diving (e.g., NSS-CDS, the National Speleological Society-Cave Diving section, and the NACD, the National Association of Cave Divers) are self-regulated, so that there is no means to force individuals to speak about this, or any other case.

To put things in perspective, it is appropriate to point out here that there are other branches of society, such as the military and the police, which regularly encounter similar, if not more tragic, events and, yet, cannot choose to avoid speaking about the issues. These branches are, of course, not self-regulated but rather are regulated by the corresponding governing agencies. As a result, acci-

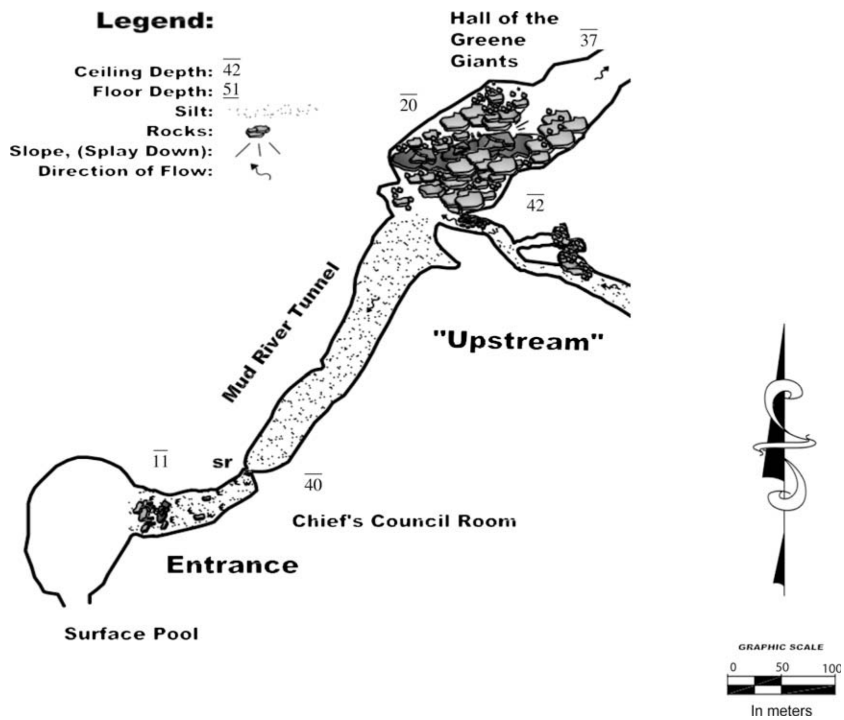

Fig. 3. A close-up map of the entrance to the Indian Springs cave system and the first few hundred meters. Depth is given in meters. The passage that was blocked during the event in question is marked as Squaws Restriction (sr). It is situated about 70 m to the right of the entrance from the Surface Pool (to the cavern) and is roughly 40 m deep. (Map adapted with permission from a larger, National Speleological Society map.)

dents are more rigorously investigated, no participant can avoid speaking about a case and, consequently, there is probably a much better understanding of accidents and how to avoid them.

1.1. The event

The accident in question occurred in 1991 prior to the establishment of the International Underwater Cave Rescue and Recovery (IUCRR, www.iucrr.org) so there is no official analysis available to the public other than the original report written by Gavin (1991), some of which is summarized below in the next two paragraphs.

The dive was the first in a series of planned exploration dives. The dive plan consisted of a 40 min transit at a depth of no more than 45 m while breathing an EAN 27 travel mix (27% oxygen instead of the normal 21%), a descent and exploration to no more than 90 m using trimix 14/44 (14% O_2, 44% He, balance N_2), followed by a return 40 min transit to exit the cave. The 90 m deep working phase of the dive was expected to last 20–25 min. The 45 m penetration and exit was done using two ‘‘stage" bottles (i.e., bottles that were carried by the divers merely for the entrance and exit and were left behind when not in use), whereas the 90 m deep portion was accomplished using back mounted doubles.

The dive went almost exactly according to plan during the penetration. Bill Gavin and Parker Turner began their exit at 63 min into the dive. They reached their staged nitrox bottles in 2–3 min, began breathing them, and did not use their back mounted doubles again until they later encountered the obstruction that caused the accident at what is known as the ‘‘Squaws Restriction" (Figs. 2 and 3). There was a distinctive arrow marker at the upstream/downstream junction, which is about 150 m from the entrance. As this arrow came into view, they estimated that their bottom time was going to be somewhere between 105 and 110 min. They made the left turn at this arrow and immediately noticed that the visibility in the cave had decreased. The floor was completely obscured by billowing clouds of silt, but the line was still in clear water near the ceiling. As they got closer and closer to the entrance, the visibility became progressively worse. Finally, they had to stop using the diving propulsion vehicles (DPVs) and swam while maintaining physical line contact.

When they arrived to the point where the restriction should have been, the line disappeared into the sand on the bottom of the cave. They attempted to pull the line out of the sand, but reached a point where it was buried too deep. Visibility in this area at that time was 30 cm or less. Closer to the exit there were two lines running parallel in the cave. They tried following both of them, but each time reached a point where the line could not be pulled out of the sand that had covered it. Ultimately, Gavin somehow managed to exit the blocked cave by removing debris and pulling himself out but Parker run out of gas and drowned. (By the time that he was out of the cave, Gavin had almost no gas left in his tanks.) This is the description of the accident according to Gavin.

There are numerous blogs that discuss the case primarily because it is so unique. Bill Gavin also wrote one of these interesting blogs. In that blog he stated his suspicion that divers present in the cavern (not cave) might have inadvertently caused the collapse and associated mudslide that blocked the exit. His reasoning was that, otherwise, the chance that such a collapse would occur exactly at the same time that the divers were in the cave is miniscule. Here, we place his suspicion on a firmer ground by suggesting an actual physical process (‘‘cave resonance") that could lead to such an outcome.

As is typical with issues of such nature, there is variability in the anecdotal descriptions of what actually happened even among those that were present in the cavern when the incident occurred. Among those descriptions, there is one alluding to exhaust bubbles released by divers decompressing in the cavern (for decompression issues see e.g., Wienke, 1991, 1992, 2009). According to this description, the resulting bubbles dislodged a large amount of sediment up in one of the solution tubes next to the ceiling, which cas-

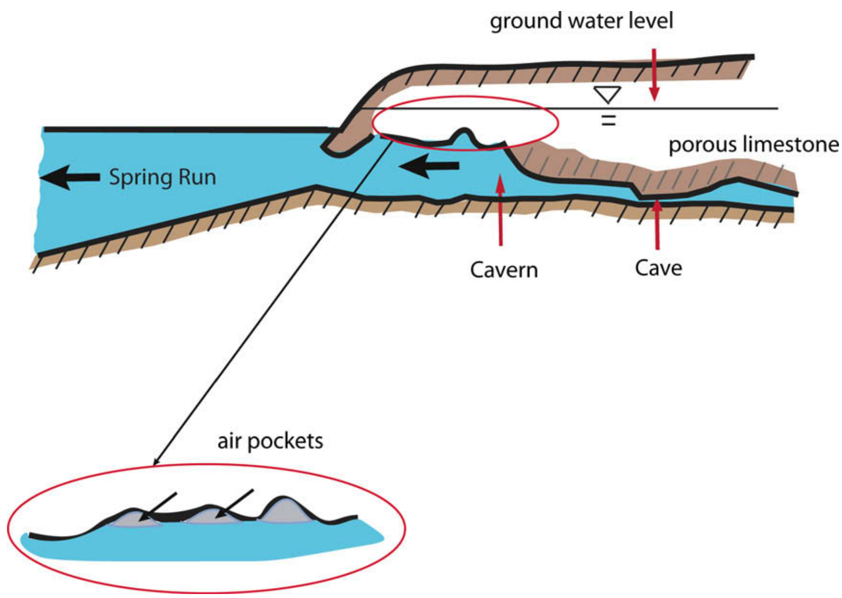

Fig. 4. A cross-section of a typical submerged cavern with an attached cave in a karst aquifer. This is the general structure of a submerged cave in Florida, not necessarily of the Indian Spring cave, which is shown in Fig. 3. Note the air pockets near the ceiling of the cavern.

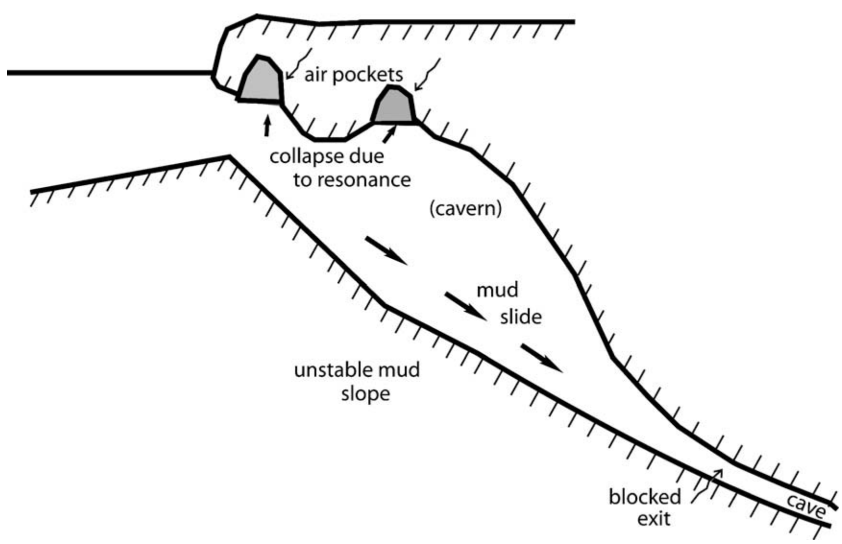

Fig. 5. Schematic diagram of our proposed cave collapse process in Indian Spring, Florida. While two divers were still in the cave beyond what is marked in Figs. 2 and 3 as the Squaws Restriction (sr) passage (lower right in this figure), divers in the cavern (upper left) generate resonance in the air pockets above them. This causes the fall-out of broken rock from the ceiling onto the steeply sloping sediment on the bottom (slope > 1:2). In turn, this impact generates a mud and sediment slide that rushes to the right and blocks the passage.

caded down onto the sediment slope, causing it to slump and plug the Squaws Restriction situated downhill (Fig. 3). One person described it as if it ''looked like someone emptying a trash dumpster from the ceiling", and continued until the visibility was obscured in the basin. As we shall see, this fits very well with the resonance mechanism that we propose here (Figs. 4–6). It is worth mentioning here in passing that a large-scale avalanche of sediment resulting from a weight dropped on the sediment slope (leading to the cave) during the US Cave Expedition of the Wakulla Springs in the late 1980's was reported by Stone (Stanton, personal communication). A similar event resulting from divers digging for art effects is described in Burgess (1999). Wally Jenkins also reported a similar event associated with dropping heavy objects on the slope within the Wakulla in the late 1950s. Next, we shall briefly review the ideas behind some well-known cases of resonance, which will serve as an introduction to the new resonant case presented here.

1.2. Resonance

Many processes in nature are subject to resonance. The simplest case is that of a swing, which goes higher and higher when pushed at the right frequency. Another example is that of a tidal resonance, which occurs when the tide excites one of the resonant modes of the ocean. The effect is most striking when a continental shelf is about a quarter wavelength wide. Then an incident tidal wave can be reinforced by reflections between the coast and the shelf edge, producing a much higher tidal range at the coast (e.g., Garrett, 1972). Famous examples of this effect are the Bay of Fundy, where the world's highest tides are found, and the Bristol Channel. Large tides due to resonances are also found on the Patagonian Shelf and on the N.W. Australian continental shelf.

In mechanics and construction, a resonance disaster describes the destruction of a building or a technical mechanism by induced vibrations at a system's resonance frequency, which causes it to

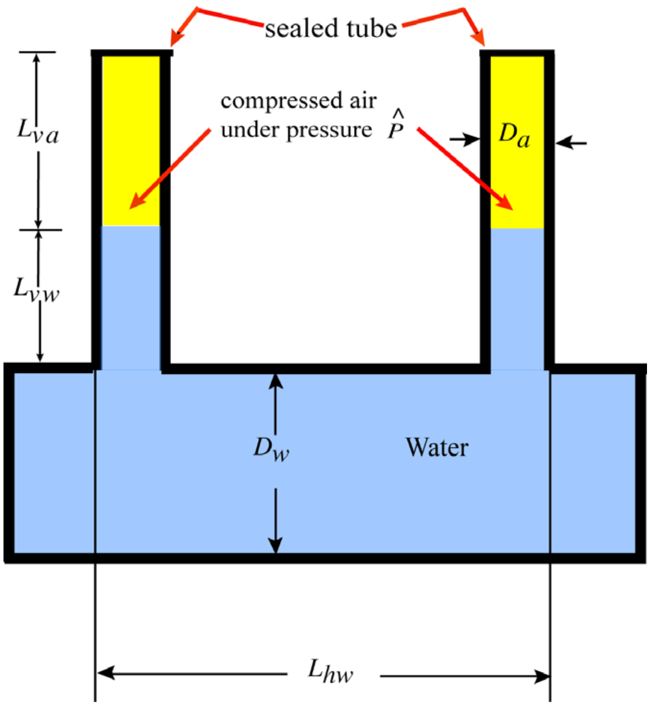

Fig. 6. The simplified *U*-tube model. The horizontal distance between the vertical components of the tube is L_{hw}, the length of the vertical compressed-gas-filled tube is L_{va} and the length of the vertical water-filled portion of the tube is L_{vw}. The air chambers pressure is $\hat{P}$; D_a and D_w are the diameters of the vertical and horizontal components. In most realistic situations $D_w \gg D_a$ but this condition is not necessary for the calculation.

oscillate. Periodic excitation optimally transfers to the system the energy of the vibration and stores it there. Because of this repeated storage and additional energy input the system swings ever more strongly, until its load limit is exceeded. The dramatic, rhythmic twisting that resulted in the 1940 collapse of "Galloping Gertie," the original Tacoma Narrows Bridge, is sometimes characterized as a classic example of resonance; however, this description maybe misleading. The catastrophic vibrations that destroyed the bridge were probably not only due to simple mechanical resonance, but due to a more complicated oscillation caused by interactions between the bridge and the winds passing through its structure – a phenomenon known as aeroelastic flutter (see e.g., Jain et al., 1996). There is also an interesting aircraft resonance case involving runway smoothness (and its associated wavelength) resonating with the length of the aircraft. For other issues involving resonance the reader is referred to: Alex and Craik (1971), Alexeev and Gutfinger (2003), Chester (1964), and Goldshtein et al. (1996). These articles do not, by any means, summarize what is known on the subject but they do provide some additional information.

1.3. Content

This paper is organized as follows. In Section 2 we present the new model and its governing equation. The inviscid oscillatory and resonating solution to the new physical system that we consider is given in Section 3. In Sections 4 and 5 we present the deep cave limit and the highly viscous limit. A generalized solution that includes some frictional representation is discussed in Section 6. A discussion and summary are given in Section 7.

2. Model and governing equation

Consider the *U*-tube shown in Fig. 6. In contrast to the classical *U*-tube problem, our new resonant problem contains two narrow vertical tubes capped at their tops, which represents the upper regions of the cave where the compressed air accumulates. Their diameter is D_a, their corresponding cross-sectional area is A_a, their total length is L_v, and the lengths of their components that are filled with air and water (respectively) are L_{va} and L_{vw}, respectively. The thick horizontal tube (below) representing the cave (or cavern) has a diameter D_w, much larger than that of the vertical tube D_a, and a length L_{hw}. The tubes diameter ratio represents the actual situation in nature but, as it turns out, it has no bearing on the calculations presented here. The corresponding cross-sectional area of the vertical tube is A_w and its wet length is L_{hw}. The excess pressure (i.e., the pressure above the atmospheric pressure) in the air-filled components depends on the cave's depth and is denoted by $\hat{P}$.

This new *U*-tube model is adopted as a means of representing resonating flows that are superimposed on the usual one-dimensional (horizontal) flow in the cave. This modeled resonating flow, which is induced by the air pockets, is limited to the region between the pockets, so the lower part of the modeled tube is taken to be blocked on the two sides, forming a *U*-tube.

When the water level in the right vertical tube (shown in Fig. 6) is elevated an arbitrary infinitesimal distance η above its neutral position there are two restoring forces. The first is the familiar weight of the displaced water $\rho_w g A_a \eta$, where ρ_w is the water density. The second is the new not-so-familiar force due to the incremental increased pressure in the air-filled section, $\hat{P}\eta/L_{va}$ (derived from the linear gas law, $\hat{P}\hat{V} = \text{const.}$, where $\hat{V}$ is the volume of the air pocket). Fortunately, this new force (i.e., the incremental increased pressure times the cross-sectional area A_a) turns out to be linear. In the absence of friction (i.e., the inviscid limit), the sum of these two forces causes the fluid to accelerate (in both the horizontal and vertical tubes) in response to the initial perturbation (e.g., increase) of the water level in the right vertical tube.

In the vertical tubes the (time-dependent) acceleration is $d^2\eta/dt^2$. Conservation of mass implies that in the horizontal tube the acceleration is much smaller, $(d^2\eta/dt^2)A_a/A_w$, because the velocity, $d\eta/dt$, is also much smaller $[(d\eta/dt)A_a/A_w$ since the mass flux has to be the same in horizontal and vertical tubes]. Assuming that the fluid velocities are uniform within each cross-section and neglecting the corners of the *U*-tube, the inviscid governing equation ($F = ma$) can be written as,

$$\rho A_a(L_{vw} + L_{hw})\frac{d^2\eta}{dt^2} + (2\rho g A_a + 2\hat{P}A_a/L_{va})\eta = C_1 \cos \omega t + C_2 \sin \omega t. \quad (1)$$

Here, the first term on the left hand side corresponds to the familiar acceleration of the water in both the narrow and thick segments of the *U*-tube. Interestingly, it turns out to be independent of the cross-sectional area of the thick tube, A_w, because, although the cross-sectional area is large there, the velocity there, $\frac{A_a}{A_w}(d\eta/dt)$, is small so it compensates for the increase in mass associated with the increase in area. (This implies the counter intuitive result that the ratio of the tubes diameters does not enter the problem.) The second term is the restoring force, which now consists of both the familiar gravitational pull and the new not-previously discussed force associated with the *compressed* air in the pockets. Luckily, this new term is linear. The terms on the right hand side represent the (known) periodic forcing associated with divers releasing air that further accumulates in the closed sections of the tube. (Given the concave nature of the cave's ceiling, this additional air does not have to be released directly into the air chambers. For most caves, air released nearby will slide along the slanted ceiling and will ultimately reach the highest point representing the top of the tubes.) When the terms on the right hand side are zero, the solution of (1) is a harmonic oscillations solution. We shall refer to this state as the "free state" as it is not subject to any outside forcing. Interestingly, the inclusion of the new enclosed (air-pressurized) sections of the tube on top does not change the mathematical nature of this solution. All they do is make the restoring force larger. When the diameter of the tube is uniform everywhere and the geometry is purely horizontal (i.e., no vertical tubes) and there is no water (so the action of gravity is eliminated) the new problem reduces to the familiar forced oscillations in a horizontal tube discussed in many scientific articles and textbooks (see, for example, Alexeev and Gutfinger, 2003; Goldshtein et al., 1996 and the references given therein). Another highly idealized circumstance is obtained in the free state case (i.e., $C_1 = C_2 = 0$) when the tubes are not sealed (i.e. $\hat{P} = 0$). Here the new problem reduces to the familiar oscillations of gravity waves (see e.g., Lamb, 1945).

3. The inviscid solution

In this no-viscosity limit represented by (1), the restoring force is the only force available for accelerating the fluid. In contrast, we shall see in Section 4 that, in the high viscosity limit, frictional forces along the boundaries oppose it and can be so large that they balance it altogether so that the fluid does not accelerate.

Assuming an inviscid solution of the form, $\eta = A_{m1} \cos \omega t + A_{m2} \sin \omega t$, we get from (1),

$$A_{m1} = C_1/(\beta - \omega^2\alpha); \quad A_{m2} = C_2/(\beta - \omega^2\alpha),$$

where $\alpha = \rho A_a(L_{vw} + L_{hw}); \quad \beta = 2\rho g A_a + 2\hat{P}A_a/L_{va}$.

We see that, regardless of the choices for the constants C_1 and C_2 (representing the strength of the forcing), the amplitudes A_{m1} and A_{m2} go to infinity (i.e., resonance) when $\omega^2 = \beta/\alpha$. As in other resonance cases, this frequency is also the frequency of the natural oscillations, i.e., the frequency of the free state ($C_1 = C_2 = 0$). Just like a swing is forced higher-and-higher when pushed at the same frequency as its natural oscillation frequency, so are the oscillations in the *U*-tube.

Accordingly, the period of the forcing leading to a resonance is found to be,

$$T = \frac{\sqrt{2}\pi}{\left\{g/(L_{vw} + L_{hw}) + \hat{P}/\rho L_{va}(L_{vw} + L_{hw})\right\}^{1/2}}. \quad (2)$$

When the forcing is at the above period, the pressure at the gas-filled chambers, $\hat{P}\,\eta/L_{va}$ (where $\hat{P}$ is the undisturbed pressure), goes to infinity (i.e. A_{m1} and A_{m2} become infinitely large). It is this infinite increase in pressure that we argue might have caused the collapse. Note that, practically, η ranges from almost zero in the beginning of the resonance process to the full extent of the gas pockets when resonant takes hold. For most Florida caves, this maximum length might be as high as several meters.

We shall now consider two examples (Fig. 7) associated with cave dimensions typical for caves in Florida. The first example deals with the case where the ceiling of the cavern/cave is just below the water elevation in the spring run (so that $\hat{P} \sim 0$, implying that the pressure changes are negligible compared to gravity) and the combined length of the cavern/cave and vertical tubes is 10 m. For these values, the period is about 4 s, which is of the same order as the typical time elapsing between two consecutive breaths of a typical diver. As a second example, suppose that the cavern/cave ceiling is at 10 m depth, the height of the air-filled chambers L_{va}, (which, as mentioned, are not necessarily fully, or even partially, visible) is 5 m, and the combined length is 40 m. Under such conditions, the period *T* is about 6 s, which is also comparable to the time elapsing between two consecutive breaths. We see, therefore, that the forcing period corresponding to resonance

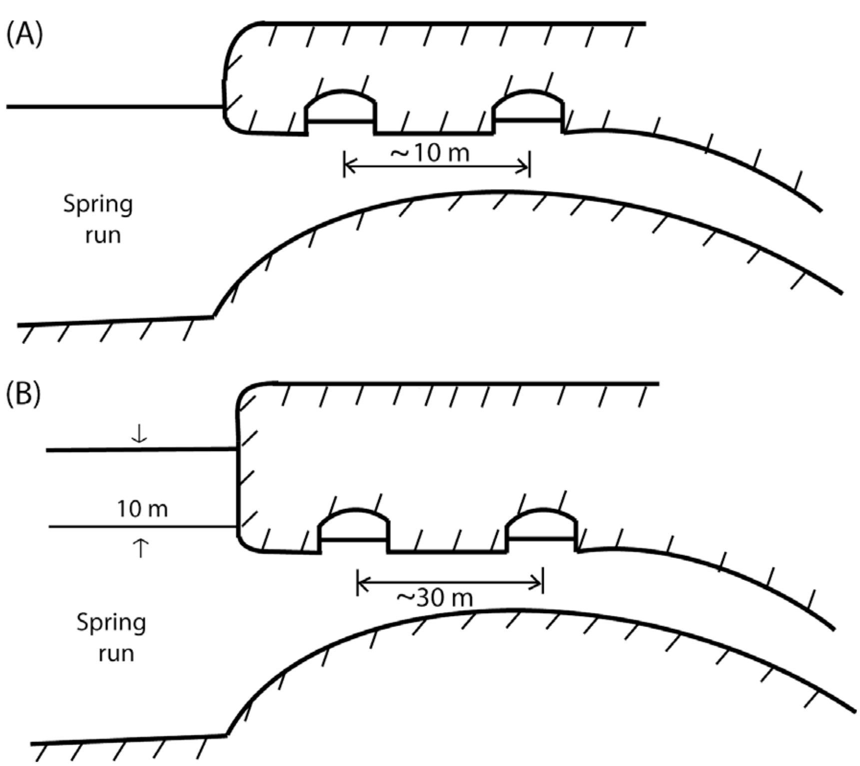

Fig. 7. Schematic diagrams of the two examples given in the last part of Section 3. In the first example (top), a cave whose air pockets are roughly at the same level as that of the water in the open spring pool is shown. In this case, the distance between the two pockets is 10 m and, due to the shallowness of the cavern, the initial pressure in the pockets is very small. In the second example (bottom), the pockets are 10 m deep and the distance between the pockets is 40 m. The resonance periods for these two examples are 4 and 6 s (respectively), periods comparable to the time elapsed between two diver's breaths. This does not mean that every cave can be subject to resonance but it does mean that the conditions for resonance can be met in many caves.

(i.e., the pressure in a typical cave becomes infinitely large) is comparable to the natural breathing period of divers in the cave. It is hard to tell today where, relative to these two examples, the Indian Spring was in 1991 because the pre-collapse ceilings then was not the same as the ceiling today. However, it makes sense to assume that it was somewhere between those two examples.

4. The deep cave limit

It is interesting to note that for deep caves satisfying $\hat{P}/\rho g L_{va} \gg 1$, the gravitational restoring force is negligible compared to that exerted by the compressed gas in the chambers. This is because the pressure in the cavities is so great that their associated restoring force is much stronger than the gravitational pull. For example, in a 50 m deep cave with 1 m long vertical air-filled cavities the force of the compressed air is 50 times larger than the gravitational pull. The resonance period in this case is short and for a 50 m long tube, the period is about 1.5 s. When the length of the cavities is short ($L_{va} \to 0$), the resonance period goes again to zero because the pressure in the chambers is so large that the oscillations occur instantly.

5. The high viscosity limit

Frictional problems in turbulent flows are hard to analyze and solve for because the forces are nonlinear in the sense that they are proportional to the square of the velocity, not the velocity itself. For simplicity, we shall first consider the free state and focus on the limit where the pressure gradient is entirely balanced by the frictional forces. Namely, in contrast to the inviscid case where the pressure gradient was entirely balanced by acceleration, in the present case, the pressure gradient is entirely balanced by frictional stresses along the boundaries of the tube and the acceleration is zero. Here, once the height of the water on the right hand side was disturbed and elevated an infinitesimal initial distance $\hat{\eta}$, it will take the system infinitely long time to drain the excess water on the right hand side. An oscillatory state will not be reached because the frequency is infinitesimally small (i.e., $T \to \infty$). At each moment in time the pressure gradient will be balanced by friction, which comes about through stresses long the conduit walls. Even when the friction plays a much smaller role, the resonant flow will still be altered. Under such conditions of limited frictional influence, the resonant related flow (which needs to be distinguished from the, much smaller, mean flow in the cave) will gradually decrease until it reaches zero at $T \to \infty$.

Since even the relatively slow mean flow in most caverns and caves is turbulent, it is expected that the, much faster, resonant related flow will surely be turbulent implying that there is no analytical solution even for the slowly varying frictional problem. (The Reynolds number for a cave 3 m in diameter with a mean flow of about $0.3\,\mathrm{ms}^{-1}$, is roughly 10^6 for a kinematic viscosity of $10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$. This is much larger than 2000 implying a strongly turbulent regime.) There are various empirical formulas for calculating the variables in question, all of which assume that the friction-induced energy loss is proportional to the square of the velocity. One formula that one can use is the Manning formula, which states that, in the free no-resonance case, the mean, slowly varying, velocity in the tube is:

$$V = \left(\frac{1}{n}\right)(D_m/4)^{2/3}[2\eta/(L_{vw} + L_{hw})]^{1/2}, \tag{3}$$

where D_m is the mean diameter of the tube $(D_a L_{vw} + D_w L_{hw})/(L_{vw} + L_{hw})$. Here, D_a, D_w are the diameters of the vertical and horizontal tubes respectively and n is a frictional coefficient that is $\sim O(0.01)$ in metrical units. In the beginning of the process, $\hat{\eta} = \eta(0)$, the flow is maximal because the elevation is maximal. It gradually diminishes in time and completely vanishes at $T \to \infty$. Since the period in this limiting high-viscosity case is infinity, the period of the forcing causing a resonance is infinity too.

6. Approximate frictional solution

The actual solution is somewhere in between the inviscid case (i.e., all the restoring force is applied to the acceleration) and the frictionally dominated case (i.e., the restoring force is completely balanced by friction so the net force and the acceleration are both zero). Namely, in reality, some of the restoring force will be applied to acceleration and some to friction. There is no simple way to determine the actual partitioning between the two but a reasonable approach would be to assume an equal partitioning, namely, to say that *half* of the restoring force is applied to the acceleration and half is balanced by friction. This assumption implies that the restoring force in (1) (i.e. the term proportional to η on its left side) should be multiplied by ½, which yields the approximate frictional solution,

$$T = \frac{2\pi}{\left\{g/(L_{vw} + L_{hw}) + \hat{P}/\rho L_a(L_{vw} + L_{hw})\right\}^{1/2}}. \tag{4}$$

The difference between (4) and (2) is merely in the numerator which now has an additional factor of $\sqrt{2} \approx 1.4$. This means that the two examples that we gave earlier in Section 3 (Fig. 7) are slightly modified. The first gives a period of 4 s for a tube merely 7 m long (instead of 10 m) and the second gives 6 s for 28 m long tube (instead of 40 m).

7. Summary and discussion

We presented a hypothesis regarding a new physical process in water-filled caves regularly explored by open circuit [OC, to be distinguished from closed circuit, rebreather (RB) divers that do not release bubbles] cave divers who release compressed gas into the caves. The essence of the new mechanism is resonance induced by the divers' breathing apparatus, which expels compressed gas with each breath. When the frequency of these breathes matches the frequency of natural oscillations in the cave gas pockets, the system is just like a swing pushed higher-and-higher when the pushing occurs each time that the swing is in its highest position. While the theory is clean and straightforward, its application to real caves is not so simple due to the need to examine the behavior of the gas pockets that might be partially embedded in the porous medium. Nevertheless, the results are informative indicating the possibility of resonance leading to very high pressures in the gas pockets generated along the cave ceiling.

A few final comments should be made. First, note that a resonant mechanical system involves the interplay of potential and kinetic energy, with the two being roughly constant over one oscillation period. The amplitude of the resonant wave grows as energy is supplied to the system. In our case, the energy source is the compressed gas in the divers tanks that is exhaled in each breath. (Work has to be done in order to fill the tanks under normal atmospheric pressure.) That energy is not immediately available as the diver exhales the gas because the pressure in the cave and the divers lungs is hydrostatic. However, as the bubbles rise, they gain kinetic energy in a similar fashion to the kinetic energy gained by a falling ball. The bubbles containing the exhaled gas increase the pressure in the capped chambers as they are forced into them by rising from the diver lungs up to the concave cave ceiling above. Namely, the resonance that we speak about is analogous to that of a swing hit by a falling ball each time that it is in its highest position. We expect that there will be some lateral bubbles motion but most of this lateral motion will be due to curvature of the ceiling rather than due to the flow in the cave. Regardless, when we speak about the pockets, we speak about the final position of the bubbles and it does not really matter how the bubbles get there. The only aspect that the resonance requires is that the resonance-inducing diver will be positioned in such away that hers/his bubbles accumulate in one of the pockets.

Second, a comment needs to be made on the length of time required to excite the system to a state close to resonance. (This can be calculated by dividing the total energy of the resonant oscillation by the rate of input of energy and is estimated to be several minutes.) This time should be at least of the same order as the time that a diver is present near the chambers. In the case in question, postings in the blogs stated that there were OC divers "hanging out" in the cavern area during the entire dive, particularly one diver hanging out in one spot. Given that the periodicity is only several seconds, the time involved was much larger, as required by the resonance. Note that, because divers use a broad spectrum of breathing rates, a single diver located near a tube resonating at his/her breathing rate is more likely to produce resonance than a group of divers. Namely, a group of divers will inevitably have divers with a breathing rate that does not match the natural oscillation. This will throw the system out of resonance just as a swing is thrown out of resonance when it is pushed at times other than those corresponding to its maximum displacement. Note that the two divers in question (Gavin and Parker) were much too far away (from the collapsed region) for contributing to the collapse with their *own* released gas, and that no divers in the cavern were on rebreathers.

In this context, it is useful to examine the air chambers size needed for our mechanism to work. Going back to our first example at the end of Section 3 (Fig. 7), suppose that the diver releases 2 l of air with each breath and that he/she is at 10 m depth. Upon rising to the cave ceiling, which, in this particular example, is just below the surface, the bubbles occupy 4 l. Suppose now that the diver stays around for 10 min during which she/he releases 150 breaths. This implies an increase of 0.6 cubic meters of air injected into the pockets. Clearly, this is an enormous increase for many pockets, whose initial pre-resonance size is often no more than about a single cubic foot (0.027 m^3).

A third comment should be made regarding the integrity of the air chambers. Since the cave boundaries are porous, air does not remain within the cave forever but gradually escapes upward through the porous medium. Evidently, this escape is often very slow as one sees these air pockets in almost every cave dive so this should not be an issue for the case in question. Finally, we will probably never know for certain what happened on that tragic day. Our findings do not by any means rule out the possibility of important independent instabilities in the bedrock constituting the ceiling of the cave. Such an instability (unrelated to the resonance) could have certainly been the cause of the collapse. The sole objective of the present analysis is to draw attention to the fact that resonance is a strong possibility. Namely, the results that we present here are informative indicating the possibility of resonance leading to very high pressures in the air pockets generated along the cave ceiling.

From a scientific point of view, this independent study needs to be followed by a more detailed computational fluid dynamics investigation that will take into account various details such as the exact dimensions of the cave, the actual viscosity and the surrounding rock's porosity and structure. The same can be said of laboratory experiments. While both should, in our opinion, be performed later on, neither one corresponds to a trivial extension of this study so both should be independent studies separated from the present investigation. The long-term practical implications of the present study is that it maybe necessary to classify caves according to the risks of resonance, regulate diving there more aggressively and perhaps even allow only rebreather diving (which does not produce bubbles) in some of them.

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